Profit and Population Dynamics
Bioeconomic Modeling and Real Options Analysis of a Fishery

by

Anthony Artino and Sarah Decipeda

A Research Project

Submitted in partial fulfillment of the requirements for the
Research Seminar
Modeling Projects (MATH 52039)

Under the supervision of
Dr. Morley Davidson

Department of Mathematical Sciences
Kent State University

March 2026

Abstract

In this project, we model the bioeconomic dynamics of a population of fish being harvested and the profit produced from the yield. The population follows logistic growth with harvesting, and profit is measured per unit catch. Changes to the economic portion of the model are also analyzed. The original system is adjusted to include periodic changes to cost, abrupt changes to cost, and sudden increase to demand. The periodic changes are modeled by a sinusoidal formula, as costs predictably increase and decrease throughout the season. Abrupt changes are written as a piecewise constant function to show costs jumping to higher and lower levels. The change in demand follows geometric Brownian motion to model random, realistic fluctuations. All models are written and plotted in Python to visualize results.

Contents

Introduction

The United States fishing industry generates billions of dollars and provides many Americans with jobs. In 2022, the commercial fishing industry made $183 billion in sales, and recreational fishing brought in another $138 billion [11]. The total number of jobs in both commercial and recreation amounted to 2.3 million. To maintain such an industry, measures must be taken to keep harvests at a sustainable rate and prevent populations from dying out. Intergovernmental as well as regional government agencies are responsible for setting annual catch limits to aid both economic growth and environmental preservation.

In order to determine these catch limits, the management councils of these agencies must work with scientists to find the overfishing limit. Fishing over this limit prevents sustainable reproductive rates, which then keeps the population from producing the maximum sustainable yield, or MSY. The MSY is the greatest amount that can be caught while ensuring there are enough fish to replenish the stock over time. Once the MSY is found, the council can calculate the overfishing limit. A separate committee then sets a recommended biological catch lower than the overfishing limit, accounting for scientific uncertainty in finding the MSY. The council provides the annual catch limit based on this recommendation.

Once the catch limit is set, however, there is no guarantee that it will be met in the season. The maximum economic yield (MEY) is the amount of fishing effort that brings the maximum amount of profit, and it is lower than the MSY. As the amount of fish caught increases, the cost of fishing effort increases as well. At the MSY level of effort, profits diminish, and fishing corporations lose more money than the extra sales would bring back to them. So, MEY may be lower than the catch limit, not reached. Additionally, the MEY may fluctuate throughout the fishing season. Faulty equipment, trends in seafood dining, and scarcity of resources such as fuel can affect optimal fishing levels. In this project, we will investigate how periodic changes in cost, abrupt changes in cost, and sudden spikes in demand affect the economics of fishing.

CHAPTER 1

Bioeconomic Modeling of a Single-Species Fishery

In this chapter, we will investigate the vast dynamics of biological population growth as well as economic harvesting. By examining a single-species fishery over a bounded 90-day season, we can establish a clear deterministic foundation that pairs continuous population adjustments with variable fishing effort. Rather than us trying to assume static harvesting, this model will help capture the seasonal urgency of a fishing fleet. While this baseline may assume constant environmental parameters, it will establish the entire theoretical groundwork that is quite necessary for subsequent empirical applications such as analyzing an historical catch, the effort, and even the total profit records throughout the mid-20th century fisheries.

1.1 Analysis of Bioeconomic Model

In our case scenario, our bioeconomic model is simply constructed in a total of three phases. These phases consist of defining the natural population dynamics, formalizing the economic profit and cost structures, and combining these elements all into a unified mathematical system.

1.1.1 Fish Population Dynamics and Harvesting

Whenever a fishery is completely undisturbed, what happens is that the biological biomass $f(t)$ is entirely assumed to grow logistically. This happens to reflect on an environment constrained by a strict carrying capacity, K, and is also driven by an intrinsic growth rate, r. Thus, the undisturbed growth rate is represented as:

$$\frac{df}{dt} = r \cdot f(t)\left(1 - \frac{f(t)}{K}\right) \tag{1}$$

However, a commercial fishery is an extractive environment. To model this, we must introduce an effort function, $E(t)$, which will represent the intensity of fishing over time (e.g., number of vessels deployed, traps set, or hours fished). In this scenario, this effort function is set over a bounded 90-day season, where it will remain at some fixed amount right at the very beginning and end, while it reaches its absolute maximum right at mid-season. For instance, by simply applying the overall mass-action principle, the rate of catch becomes proportional to both the current population density as well as the total effort exerted, which is adjusted by the catchability coefficient, q. Therefore, the differential equation that is describing the rate of change of the population under harvesting ends up becoming:

$$f'(t) = r \cdot f(t)\left(1 - \frac{f(t)}{K}\right) - q \cdot f(t)E(t), \quad f(0) = f_0 \tag{2}$$

Here, the first term represents the biological renewal, while the second term, $q \cdot f(t)E(t)$, represents the mortality due to the harvesting.

1.1.2 Economic Pressures: Profit per Unit Catch

For our next step, to be able to establish an economic foundation of the model, our main initial goal would be to derive an equation for the profit per unit catch. By following the system's constraints, let p represent as a fixed market price per unit of catch, and then let c represent the operational cost per unit of fishing effort. We will then begin this derivation with the basic biological extraction of the overall fishery.

The total harvest, or Total Yield $Y(t)$, is generated according to the standard Schaefer production function:

$$Y(t) = q \cdot f(t)E(t) \quad \Longrightarrow \quad \frac{E(t)}{Y(t)} = \frac{1}{q \cdot f(t)} \tag{3}$$

where q is defined as the catchability coefficient, $f(t)$ is the population of fish at time t, and $E(t)$ is the effort of fishing.

From this, we can define the Catch Per Unit Effort (CPUE), denoted as U. By using q to help represent the total catch in this context, the biological efficiency of the fleet will be:

$$U = \frac{q}{E(t)} \quad \Longrightarrow \quad \frac{E(t)}{q} = \frac{1}{U} \tag{4}$$

With the harvesting dynamics specifically outlined, we can now proceed to formulate the economic framework. Our total revenue at time t is defined as the price that is multiplied by the catch $p \cdot q$. On the other hand, our total cost is proportional to the effort that is exerted $c \cdot E(t)$. Therefore, the total profit of $\pi(t)$ becomes defined as the revenue subtracted by the cost:

$$\pi(t) = pq - cE(t) \tag{5}$$

Next, to be able to simply obtain the overall profit per unit catch, we would need to divide the total profit by the total catch q. This can be done by substituting our relationships for the Catch Per Unit Effort ($U = q/E$) where we can then factor and simplify the overall economic model by:

$$\begin{aligned} \frac{\pi(t)}{q} &= \frac{pq - cE(t)}{q} \\ &= p - c\left(\frac{E(t)}{q}\right) \\ &= p - c\left(\frac{1}{U}\right) \end{aligned}$$

Due to the total catch being equivalent to the yield ($q = Y(t)$), we can now substitute our earlier biological definition where $\frac{1}{U} = \frac{1}{q \cdot f(t)}$ as q is returning to represent the catchability coefficient. In this case, our overall equation for the profit per unit catch becomes:

$$\frac{\pi(t)}{q} = p - \frac{c}{q \cdot f(t)} \tag{6}$$

Hence, this final equation helps to demonstrate that the profit per unit catch will equal to the fixed market price subtracted by the effective extraction cost per harvested unit. Since this cost term specifically depends on the overall stock size, a decline in $f(t)$ will increase the effective cost per unit catch as it heavily compresses the profit margins as well as intensify the economic pressure on the fishery.

1.1.3 The Paired Deterministic Model

As we head into the stage of finalizing a complete model that involves the economics of fishing, we will need to first integrate our previous derivations together. For instance, by combining the biological population dynamics from our first section with the financial constraints that was established in the second section, we can now create the paired deterministic model.

This complete baseline model pairs the biological realities directly with the overall economic outcomes. Furthermore, this overall system gives us the opportunity to simply observe the continuous feedback loop where the rate of harvesting impacts the total biomass, which implies the financial returns of the fishing vessels:

$$\begin{cases} f'(t) = r \cdot f(t)\left(1 - \frac{f(t)}{K}\right) - q \cdot f(t)E(t) \\ \frac{\pi(t)}{q} = p - \frac{c}{q \cdot f(t)} \end{cases} \tag{7}$$

In this paired system, our two equations work together to describe the overall state of the fishery at any given time t:

- The first equation represents the biological component which determine the overall continuous rate of change of the fish stock $f(t)$ to both its natural logistic growth as well as its extraction rate of the fishing fleet.
- The second equation represents the main economic component which calculates the total profit per unit catch $\frac{\pi(t)}{q}$ as it's solely based on the extraction efforts.

Overall, the key fundamental link between these two equations is the population size, $f(t)$. Due to the economic profit basically relying on the biological stock, in our case, something like a depleted ocean would result in an unprofitable fleet. By being able to unite these concepts into Equation (7), we have created a successful mathematical model for the economics of fishing. Hence, this paired system represents as a complete theoretical foundation where it can be applied to specific numerical parameters as well as our bounded seasonal effort variables for computational simulation.

1.2 Numerical Application of the Baseline Model

To be able to demonstrate how this paired bioeconomic system tends to operate in full practice, suppose we are observing a commercial fishing fleet operating in the Gulf of Alaska. In this case scenario, the fleet is targeting a population of Pacific Halibut over a strictly bounded 90-day summer season. For instance, let us assume that the local marine biologists are estimating the environment's carrying capacity to be $K = 11,715$ units of biomass, with an intrinsic biological growth rate of $r = 0.05$. As the opening day approaches, the initial recorded halibut stock is at $f_0 = 9,080$, as the fleet's historical catchability coefficient is recorded to be $q = 0.001$. Suppose the wholesale market price per unit catch is $p = \$40$ on the economic side, while the operational cost to maintain the vessels and crew runs at $c = \$150$ per unit of effort.

In a realistic commercial fishery, the seasonal fishing effort $E(t)$ can be modeled by using a sinusoidal function. From here, the baseline minimum effort at the season's boundaries will be $E_{min} = 5$ as it would peak at the midseason up to a maximum effort of $E_{max} = 50$:

$$E(t) = 5 + 45 \sin\left(\frac{\pi t}{90}\right) \tag{8}$$

Substituting our specific Gulf of Alaska parameters into our biological model (Equation 2) leads to the specific initial value problem for this fishery:

$$f'(t) = 0.05 \cdot f(t)\left(1 - \frac{f(t)}{11715}\right) - 0.001 \cdot f(t)\left[5 + 45 \sin\left(\frac{\pi t}{90}\right)\right] \tag{9}$$

Example 1: Initial System State at the Opening Season (Day 0)

Problem: Suppose, at the opening of the season ($t = 0$), your task is to calculate the initial fishing effort, the instantaneous rate of change of the fish population ($f'(0)$), and the initial profit per unit catch.

Solution:
We will first begin this problem by evaluating the effort function at $t = 0$ to help establish the baseline fleet activity:

$$\begin{aligned} E(0) &= 5 + 45\sin\left(\frac{\pi(0)}{90}\right) \\ &= 5 + 45(0) = 5 \text{ units of effort} \end{aligned}$$

Next, we will need to find the initial population growth rate by substituting $t = 0$, $E(0) = 5$, and our initial stock $f(0) = 9080$ into Equation (9):

$$\begin{aligned} f'(0) &= 0.05(9080)\left(1 - \frac{9080}{11715}\right) - 0.001(9080)(5) \\ &= 454(1 - 0.77507) - 45.4 \\ &= 454(0.22493) - 45.4 \\ &= 102.12 - 45.4 = 56.72 \text{ units/day} \end{aligned}$$

Due to the natural logistic growth being (102.12 units/day), we can see that this outweighs the minimal harvesting mortality of (45.4 units/day). Thus, the population appears to be growing during the opening days of the season.

As for our final step, we will need to calculate the initial profit margin by substituting the known variables into Equation (6):

$$\begin{aligned} \frac{\pi(0)}{q} &= 40 - \frac{150}{0.001(9080)} \\ &= 40 - \frac{150}{9.08} \\ &= 40 - 16.52 = \$23.48 \text{ profit per unit catch} \end{aligned}$$

Example 2: System Dynamics at the Midseason Peak (Day 45)

Problem: As the midseason ($t = 45$) approaches, the fleet will reach its maximum effort. Assuming that the numerical integration will determine the population has dropped to $f(45) \approx 7000$, calculate both the instantaneous population growth rate and the new profit margin.

Solution:
Similar to the first example, we can evaluate the effort function at the $t = 45$ midseason mark that

leads to the maximum capacity:

$$E(45) = 5 + 45\sin\left(\frac{45\pi}{90}\right) = 50 \text{ units of effort}$$

With the peak effort being established here, we can then substitute our new value $E(45) = 50$ and the degraded population $f(45) = 7000$ directly into our growth and profit equations:

$$\begin{aligned} f'(45) &= 0.05(7000)\left(1 - \frac{7000}{11715}\right) - 0.001(7000)(50) \\ &= 350(1 - 0.59752) - 350 \\ &= 350(0.40248) - 350 \\ &= 140.87 - 350 = -209.13 \text{ units/day} \end{aligned}$$

$$\begin{aligned} \frac{\pi(45)}{q} &= 40 - \frac{150}{0.001(7000)} \\ &= 40 - \frac{150}{7} \\ &= 40 - 21.43 = \$18.57 \text{ profit per unit catch} \end{aligned}$$

Analysis: At this point, we can see here that the massive effort adjusts the harvesting mortality up to 350 units/day as its heavily overpowering the biological renewal and causing the population to drop down rapidly. Hence, this depleted stock seems to create an immediate strain on the economic efficiency as it reduces the overall profit margin.

Example 3: Economic Impact on the Last Day of the Season (Day 90)

Problem: At the end of the season ($t = 90$), the fleet will return to the baseline effort. Lets Assume that the midseason peak completely depleted the fishery to $f(90) \approx 4000$, calculate both the final population growth rate and the concluding profit margin.

Solution:
We will need to first evaluate the final effort function at $t = 90$ to help confirm that the fleet has scaled down to its baseline:

$$E(90) = 5 + 45\sin\left(\frac{90\pi}{90}\right) = 5 \text{ units of effort}$$

Substituting $E(90) = 5$ as well as the depleted stock $f(90) = 4000$ into our paired system will produce:

$$
\begin{aligned}
f'(90) &= 0.05(4000)\left(1 - \frac{4000}{11715}\right) - 0.001(4000)(5) \\
&= 200(1 - 0.34144) - 20 \\
&= 200(0.65856) - 20 \\
&= 131.71 - 20 = 111.71 \text{ units/day}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\pi(90)}{q} &= 40 - \frac{150}{0.001(4000)} \\
&= 40 - \frac{150}{4} \\
&= 40 - 37.50 = \$2.50 \text{ profit per unit catch}
\end{aligned}
$$

Analysis: Due to the fleet effort completely dropping back to 5, the mortality tends to decrease as it finally allows for the biological stock to enter a recovery phase with a net positive growth rate. However, even though the effort appears to be quite identical to Day 0, the overall depleted population of (4000) seem to cause the extraction cost to completely escalate as it nearly erases the profit margin altogether.

1.3 Computational Results and Discussion

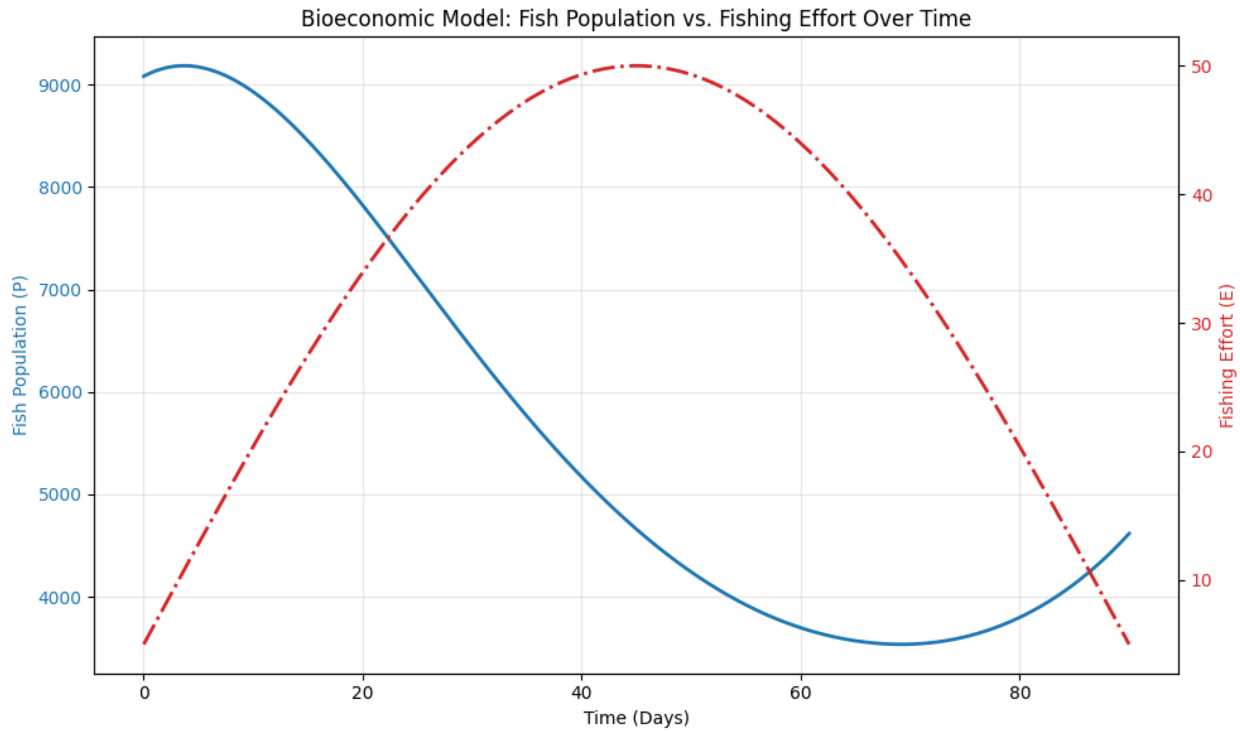

Figure 1.1

From the graph, we can see the inverse relationship between the population of fish and level of fishing effort over the 90-day fishing season. The curve of the effort function is as described in Introduction to Computational Science, module 7.13. The effort is constant at the beginning and end of the season, and it reaches a peak at 45 days. Towards the end of the season, as the effort level declines and less fish are harvested, we see the population pick back up. Because less efforts to harvest are being made, the stock has the opportunity to regenerate.

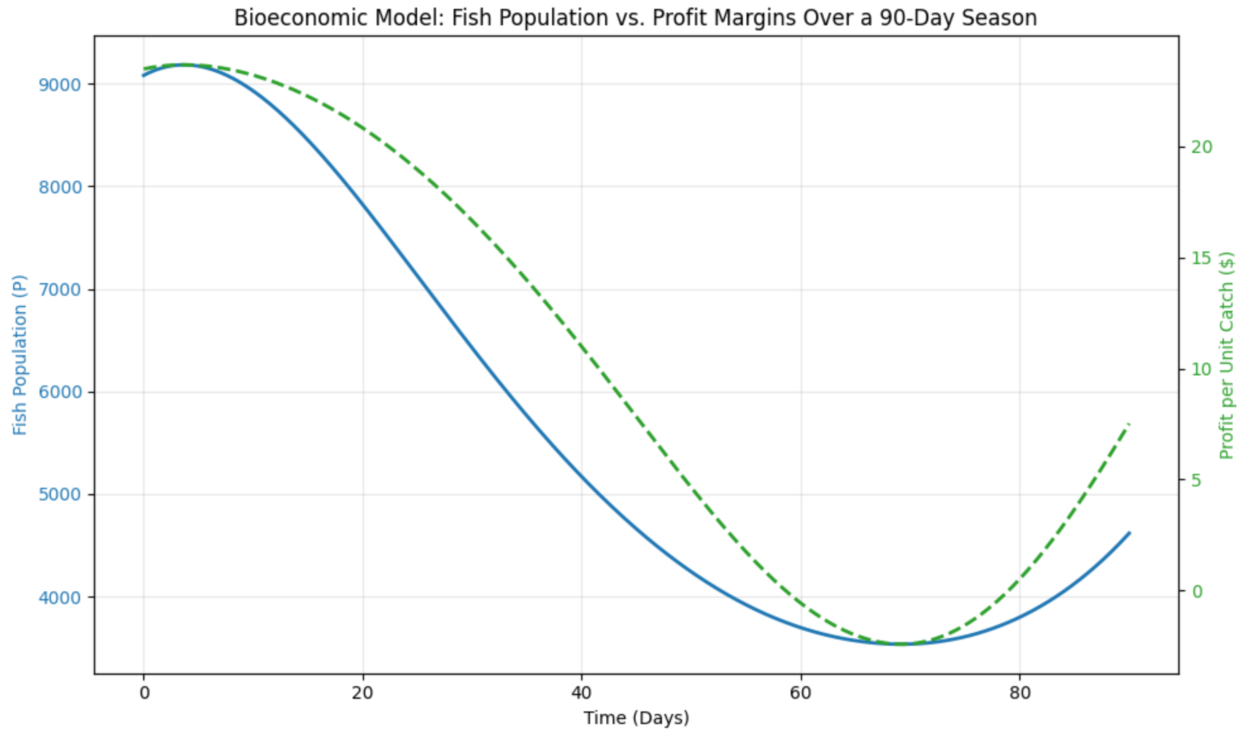

Figure 1.2

This graph compares the profit to the fish population throughout the season, and we can see they follow similar curves. Both profit and population are highest at the beginning of the season and reach their minimums around the same time. After reaching the minimum, the population begins to regrow and profits start to increase. Based on the previous plot, we see that profit is not maximized with maximum effort as it does not peak at 45 days. This occurs because the cost of the effort level offsets the amount of profit being earned.

Chapter 2

The Economic Extensions of the Bioeconomic Harvesting Model

In the last chapter, we were able to create a baseline deterministic model that helped assume the economic parameters based off of the market price and the operational cost as this remained constant throughout the 90-day fishing season. However, real-world fisheries have faced quite some dynamic economic pressures. In this chapter, we will expand our overall bioeconomic model by introducing time-dependent economic variables. Furthermore, by replacing our static constants with periodic, abrupt, and demand-driven functions, we can capture how the seasonal changes, sudden cost shocks, and market price spikes impact the biological stock as well as the financial viability of the fishing fleet.

2.1 Periodic Seasonal Costs

Our very first extension of the baseline model addresses the reality that the cost of fishing is rarely static. Due to there being all sorts of seasonal changes that occur, the cost per unit effort may follow a periodic cycle. To take this for consideration, we need to convert our cost parameter from a constant c to instead a more time-dependent periodic function of $C(t)$.

2.1.1 The Piecewise Cost Function

For us to create some smooth, repeating seasonal fluctuations over our bounded 90-day season, we will first model the cost per unit effort by using a sinusoidal formula:

$$C(t) = C_0 + A\sin(\omega t + \phi) \tag{10}$$

Now, within this function, C_0 represents the baseline average cost per unit effort as this is equivalent to our original constant c, while A represents the amplitude as this shows the maximum financial fluctuation for how far the cost will end up rising above or falling below the baseline. For us to make sure that the cost will end up completing exactly one full cyclical fluctuation over the 90-day season, ω represents the angular frequency where $\omega = \frac{12\pi}{90}$. Finally, the phase shift ϕ

allows us to align the peak costs with the specific days of the season. With the baseline start, we will assume that $\phi = 0$.

2.1.2 The Updated Paired Deterministic Model

Furthermore, with the static cost c being replaced by the time-dependent $C(t)$, we can then update our original Paired Deterministic Model. The biological extraction rate will remain quite consistent with our previous derivations; however, the profit per unit catch will be different as it must now dynamically respond to both the decline of the fish population $f(t)$ as well as the total fluctuations in operational costs $C(t)$.

As we first substitute Equation (10) into our newly established profit margin formula, this profit per unit catch will become:

$$\frac{\pi(t)}{q} = p - \frac{C_0 + A\sin(\omega t + \phi)}{q \cdot f(t)} \tag{11}$$

We can now update the paired deterministic model for a fishery experiencing seasonal cost fluctuations to:

$$\begin{cases} f'(t) = r \cdot f(t)\left(1 - \frac{f(t)}{K}\right) - q \cdot f(t)E(t) \\ \frac{\pi(t)}{q} = p - \frac{C(t)}{q \cdot f(t)} \end{cases} \tag{12}$$

Hence, this newly updated system introduces a brand new layer of economic complexity. Due to the profit per unit catch being subjected to both a declining biological population as well as an actively oscillating cost function, we will see that the overall profit margins will in fact face some compounded strain when periods of high seasonal costs directly align with periods of high biological depletion.

2.1.3 Computational Results and Discussion

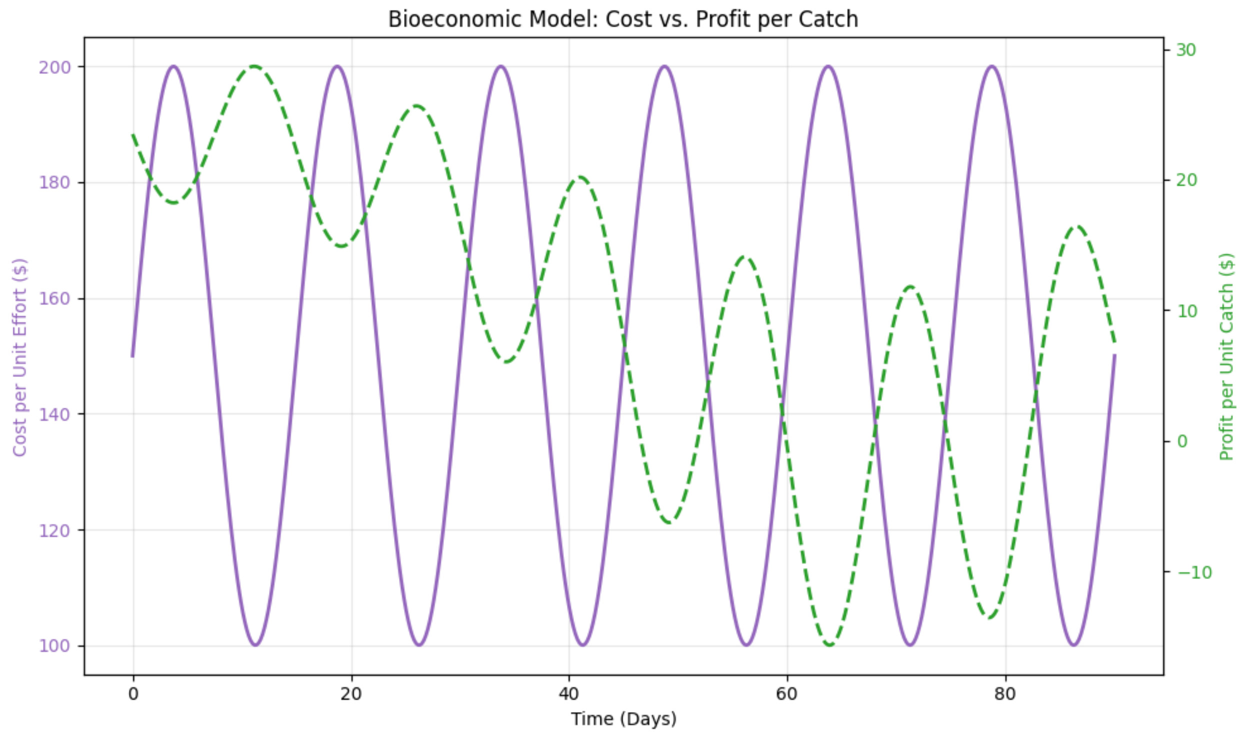

Figure 2.1

In this plot, we see the impact of predictable, seasonal costs on the profit margin. The fluctuations in cost are modeled with a sinusoidal wave and produces a similar effect in the profit curve. Clearly, profit oscillates inversely with cost, which is to be expected as higher costs reduce profit. As time goes on, the amplitude of the profit waves increase despite the size of the cost waves being constant. This tells us that the effect of seasonal costs builds up and has a more drastic impact on profit.

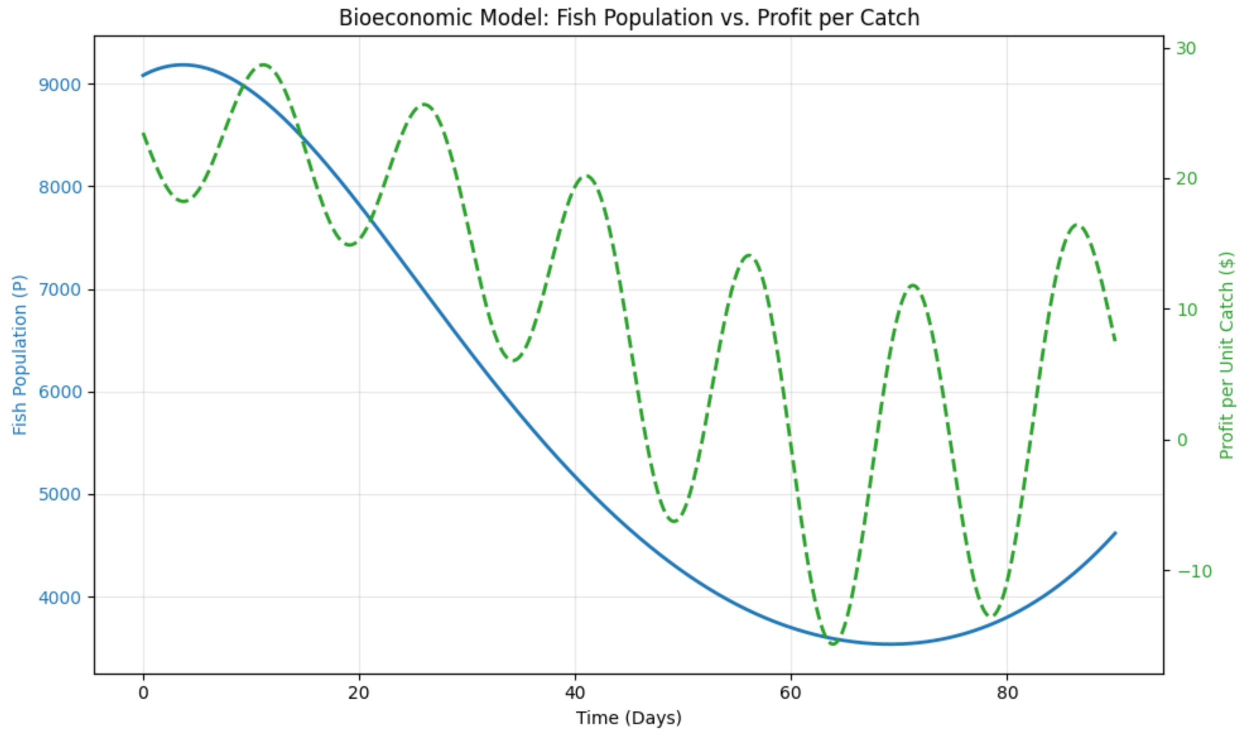

Figure 2.2

Despite the waves of the profit function, it is clear that the curve it oscillates around is similar to the curve of the original profit-population plot. The maximum is achieved early on in the season and reaches the lowest point at the trough of the wave nearest to the population minimum. Once again, the profit increases with the population, but the amplitude of the waves are still greater than earlier in the season. Thus, the effect of the cost function continues to build and have a greater impact on profit, even as profit increases.

2.2 Abrupt Cost Changes

While our previous section modeled predictable seasonal fluctuations, we must consider that real-world fisheries are subjected to sudden, unpredictable economic shocks. To model abrupt changes to the cost per unit effort, we must replace our continuous cost function with a discontinuous piecewise function.

2.2.1 Defining the Piecewise Cost Function

To accurately represent an abrupt, step-like shift in operational expenses, we define the cost per unit effort $C(t)$ across three distinct time intervals: the initial baseline period, the duration of the economic shock, and the subsequent recovery period.

$$C(t) = \begin{cases} c_0, & t < t_1 \\ c_1, & t_1 \leq t < t_2 \\ c_2, & t \geq t_2 \end{cases} \tag{13}$$

Within this piecewise function, c_0 represents the initial baseline cost per unit effort right before any abrupt economic changes occurs. The parameter c_1 represents the spiked (or dropped) cost level during the event window from t_1 to t_2. Finally, the parameter c_2 establishes the brand new cost level right after the event has finished, which has the chance to return directly to the original baseline ($c_2 = c_0$) or possibly stabilize at a new permanent baseline rate.

2.2.2 The Total Impact on the Paired Deterministic Model

For this part, we can integrate this new piecewise cost function right into our dynamic framework from Equation (12). By doing this, our profit per unit catch becomes heavily dependent on the specific time domain t.

Furthermore, due to the profit margin being inversely tied to this piecewise cost function, an abrupt spike in $C(t)$ during the midseason peak effort will cause the profit to completely plummet right away. Thus, this dynamic may cause the fishery to have many negative returns even before the cost normalizes at t_2.

2.2.3 Real-World Situations Causing Abrupt Changes

For us to further analyze this piecewise mathematical behavior, we can start making some assumptions about what types of theoretical real-world situations can trigger such sudden changes in the cost per unit effort. For instance, one common cause could be a spike in the fuel and energy. In this case scenario, due to the acts of commercial fishing fleets being so relatively dependent on diesel fuel, any geopolitical crisis or localized refinery disruption that causes fuel prices to double overnight will result in an increase of the baseline operating cost.

Another potential cause that may create a sudden cost change is regulatory inventories. For instance, if a government or environmental agency decides to implement some sort of emergency at the mid-season mark, creating restrictions could result in the fishers utilizing expensive bycatch reduction devices or forcing vessels to travel much farther away, which can create an upward shift in cost. On the other hand, government actions can also create an abrupt decrease in expenses. For example, if local authorities decide to introduce a mid-season fuel subsidy or an emergency financial grant to support the commercial fleets, what could possibly happen here is that the baseline operational cost for the vessels may end up suddenly dropping, as this would temporarily widen the profit margins as well as relieve the economic pressure on the fishery.

Acts of labor disputes can also play into effect, as there can be scenarios where perhaps dockworkers or even crew members may decide to completely stop working. If that's the case, this sort of enactment would cause all boat operators to be forced to temporarily pay much higher independent contractors, as they have to desperately keep all boats running at all times even when there could be a dispute with staff.

Finally, there potentially being infrastructure or equipment failures can also create some sort of abrupt cost changes. If this was the case, suppose a severe storm ends up destroying a local ice plant or bait supply chain. What would happen here is that fishermen may need to travel farther away to just buy off of suppliers. This could result in the suppliers charging more and there being transportation costs on the side.

2.2.4 Computational Results and Discussion

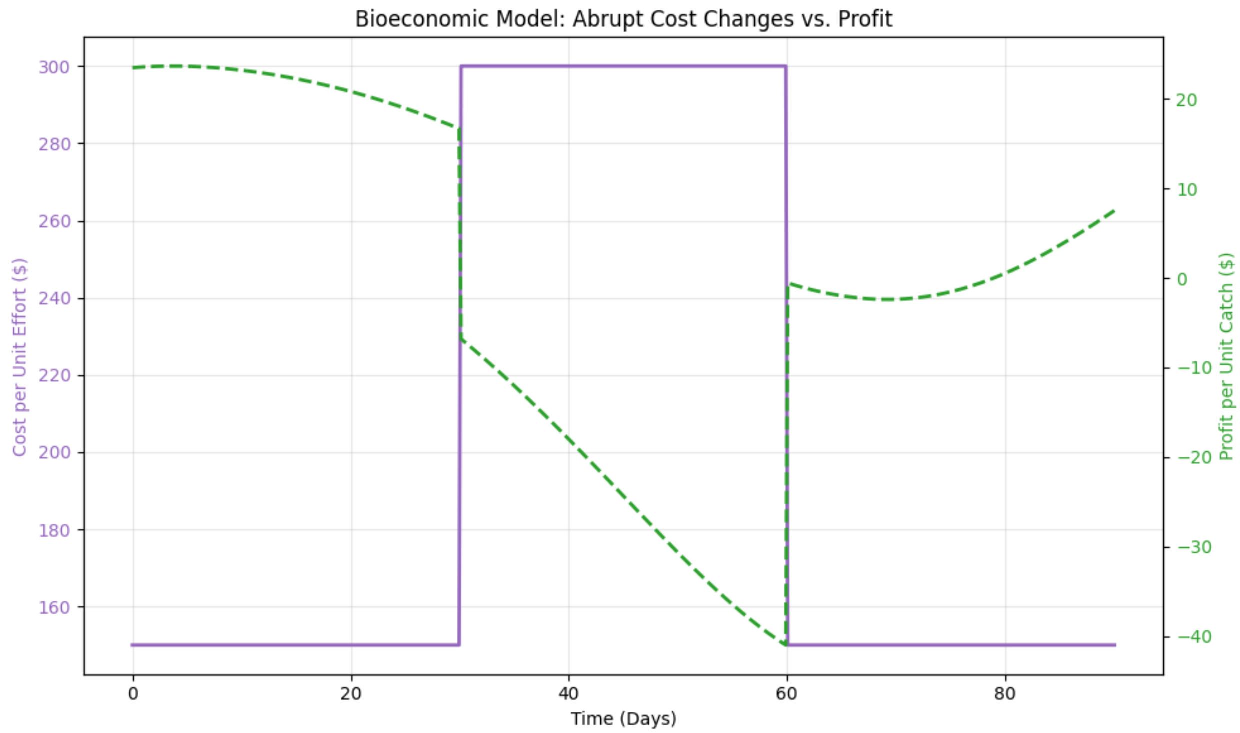

Figure 2.3

In this model, we introduce an abrupt change to the cost per unit of fishing effort. Cost is plotted as a piecewise constant function split into three intervals. At day 30, the cost doubles, causing the profit function to have a steep drop-off. During the time interval of elevated cost, profit steadily declines. At day 60, the cost returns to its original level and profit jumps up. However, it does not increase until about day 70, but it then slowly increases until the end of the season.

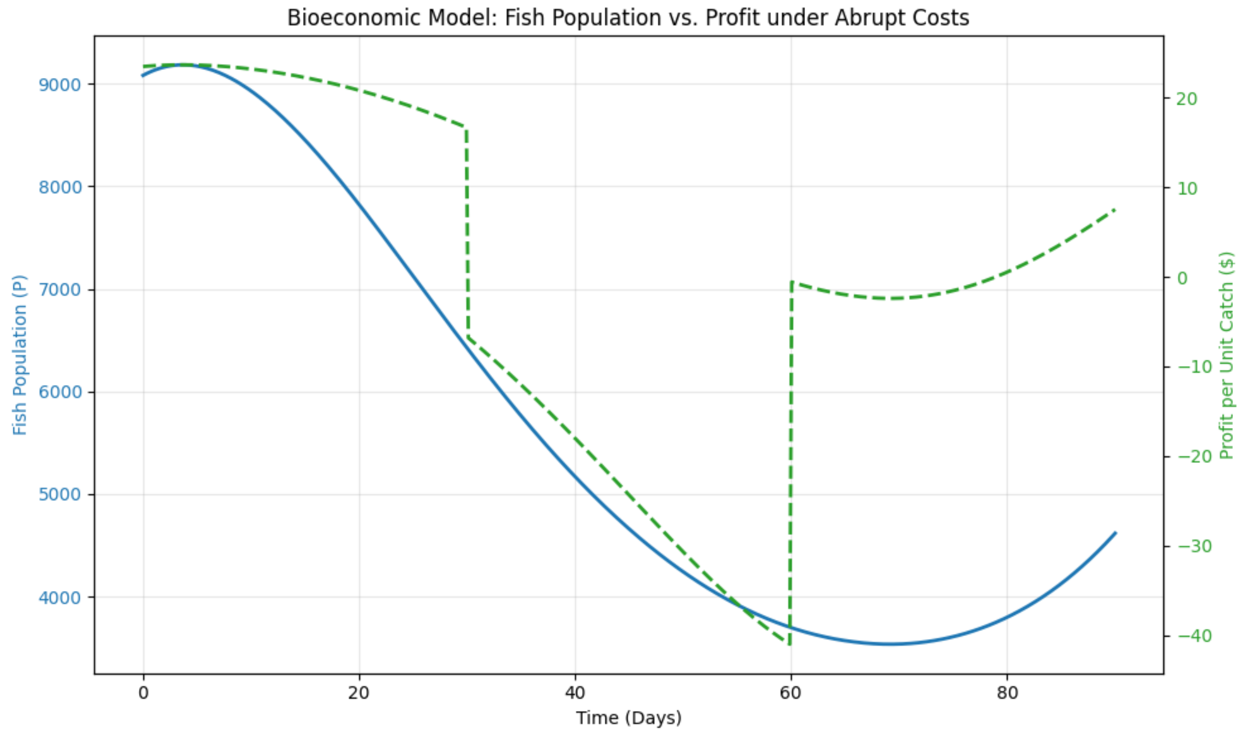

Figure 2.4

From this population versus profit plot, profit per unit catch is once again highest with the population maximum. It slowly declines as expected from the original graph, but the steep, immediate drop clearly shows where the cost was raised. The curve decreases linearly and reaches its minimum at the 60-day mark, when the cost returns to its original level. Instead of matching or even approximately matching the population minimum, the profit margin global minimum is determined primarily by this jump in cost. We still see a local minimum aligning with that of the population curve, and the increasing population once again causes profit to rise.

2.3 Sudden Demand and Price Increases

From our previous extensions, we were able to simply enhance the economic model by introducing some dynamic fluctuations to the operational cost per unit effort. On the other hand, the market price per unit catch, p, has been a static constant. But in reality, fish are market commodities which means that their value is susceptible to external market forces where there can be all sorts of sudden surges in consumer demand as well as unpredictable trading volatility. For us to be able to answer the main objective of modeling the sudden increases in demand and price, we will need to adjust our deterministic price constant to make it become a continuous-time stochastic process.

2.3.1 The Stochastic Price Function

To capture both the upward trend of the sudden demand as well as the random, erratic daily swings of a real-world market accurately, we will need to first model the price per unit catch $p(t)$ by using the Geometric Brownian Motion (GBM). Throughout financial mathematics, this is considered the standard Stochastic Differential Equation (SDE) which is used to help model commodity prices.

$$dp(t) = \mu p(t)dt + \sigma p(t)dW(t) \tag{14}$$

Within this stochastic differential equation, μ represents the drift, which assumes the expected trend (such as a steady growth in demand over the season). The parameter σ represents the volatility as it captures the randomness and erratic market shocks that will occur day-to-day. Finally, $dW(t)$ represents the Wiener process (the standard Brownian motion), as this serves as the key mathematical engine which provides the continuous randomness throughout the season.

To implement this continuous stochastic process computationally, we will need to utilize that exact analytical solution to this differential equation (derived by Itô's Calculus). This will give us an explicit price at any given time t, given an initial starting price p_0:

$$p(t) = p_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W(t)\right) \tag{15}$$

2.3.2 Real-World Situations Causing Demand and Price Spikes

There are all sorts of real-world situations that can cause sudden increases in demand and price, as we can simply analyze several external market and environmental spikes. For instance, something like natural disasters or supply shocks in other regions can be a strong indicator to abruptly reduce global supply. If there are severe weather events that occur, ecological disasters, or certain disease outbreaks that negatively affect a major competing fishing region, such as a hurricane destroying a main fishing area, it could result in domestic buyers desperately rushing to purchase local alternatives. This sudden situation can create immediate spikes in demand as well as wholesale prices for the remaining available species.

Seasonal and holiday demand is another prime example of how it can cause intense surges in seafood consumption. During major cultural events, such as Christmas or Valentine's Day, many restaurants and wholesale markets typically have a drastic increase in bulk purchases. Basically, this concentrated purchasing behavior would raise the market price due to the heightened consumer demand that occurs within a very short window.

Finally, through market speculations, there are always certain case scenarios where we could expect abrupt price increases to occur. There can be traders and wholesale distributors who are anticipating future scarcity who may decide to be proactive and purchase large quantities of fish, creating a temporary surge in demand as well as pushing the prices to be higher. Even before a tangible supply shortage actually occurs in the water, this speculative financial activity can help to simply produce abrupt price surges throughout the market, as this would perfectly mirror the stochastic volatility modeled in our Geometric Brownian Motion.

2.3.3 Computational Results and Discussion

Figure 2.5

The plots show the relationship between changes in market price and profit margin. The price is modeled using the stochastic differential equation of geometric Brownian motion to introduce random and sudden fluctuations. Clearly, the changing price has a significant impact on the profit function. In realization 2, the two curves are nearly identical, and they are extremely similar in the fifth and sixth realizations. From the other graphs, we still see that the price function determines the shape of the profit function. Though scaled differently, the minimums and maximums occur at the same time.

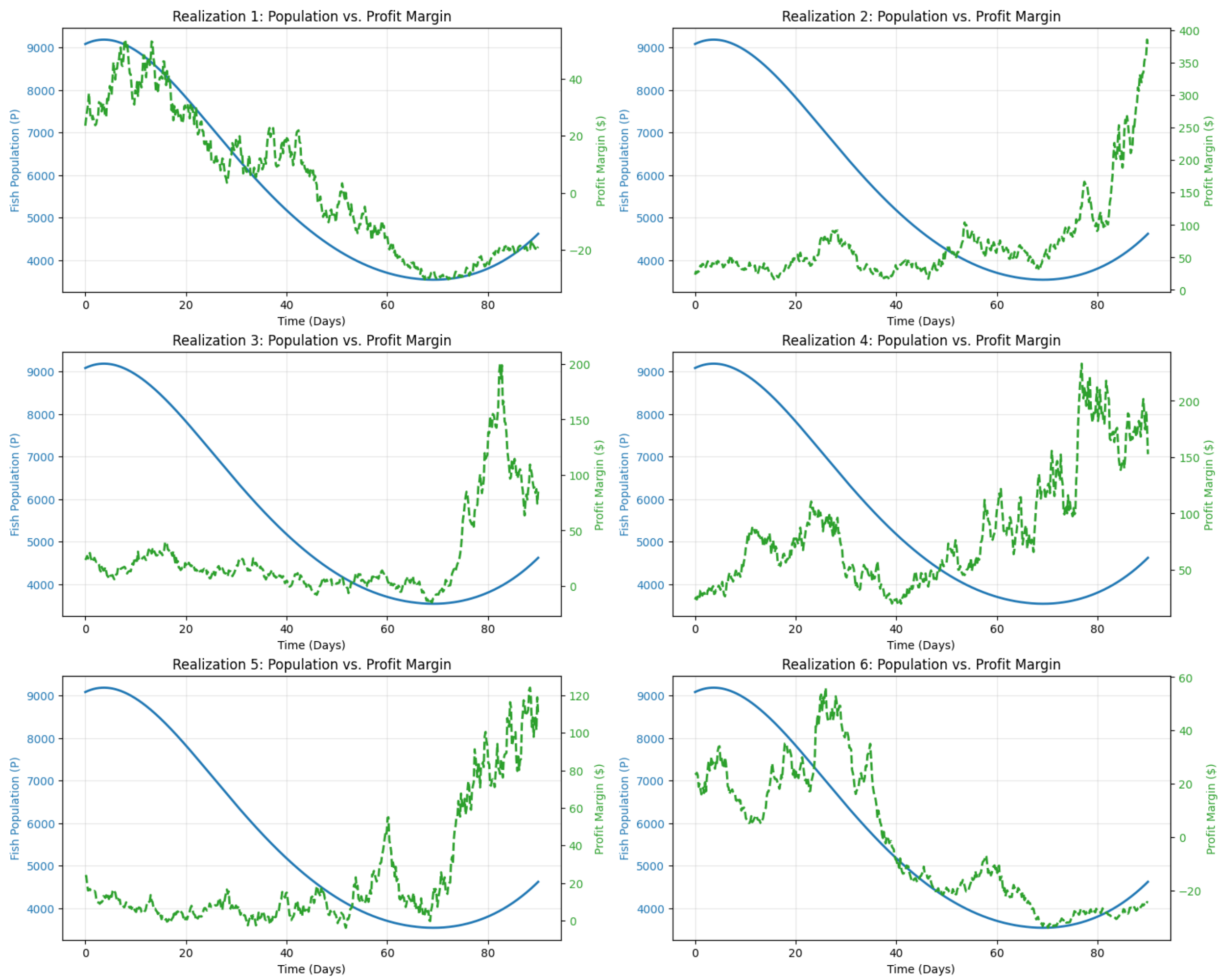

Figure 2.6

In the figures shown above, it is more difficult to see the relationship between population and profit margin compared to the previous examples. The profit margin does not necessarily reach its maximum at the same time as the population. Here, that only happens once, in the third realization. One trend we do see is an increase in profit in the last several days of the season, but the levels of increase are very different. This suggests that the market price has a much greater impact on profit than the population.

Chapter 3

The Black-Scholes Fishery

In our previous chapter, we established the baseline deterministic models and have expanded those models by using the stochastic market volatility with the geometric Brownian motion (GBM). While running these random price spikes, they have provided a realistic view of the market behavior. This has created a complex optimization problem which raises the question: If tomorrow's price is highly uncertain, should a vessel harvest today or wait? To answer this question, we want to bridge the gap between bioeconomics and quantitative finance by adapting the Black-Scholes Partial Differential Equation (PDE). For instance, we can use real options analysis, where we can mathematically value the fishery not only as a biological resource but also as a financial derivative.

3.1 The Profit Payoff Function

Throughout the financial markets, a European call option provides the holder the right to purchase an asset at a predetermined strike price. We can then apply this precise framework to our fishery. A fishing vessel usually holds a "call option" to harvest the biomass. So in this scenario, the underlying asset here is the stochastic market price per unit of fish, $p(t)$.

Unlike a traditional financial option where the strike price is basically fixed in a contract, the "strike price" for a fishing fleet represents the biological extraction cost. We can recall our profit margin derivation from Chapter 1, where the effective cost to extract a single unit of catch would be directly tied to the overall population $f(t)$. Therefore, we can define our dynamic strike price, $X(t)$, as:

$$X(t) = \frac{c}{q \cdot f(t)} \tag{16}$$

With the asset price as well as the strike price being defined, we can then establish the Profit Payoff Function, $\Pi(p, f)$. For instance, a rational fleet will only deploy its effort if the market price tends to exceed the biological extraction cost. If the market price ends up plummeting below the extraction cost, the fleet will dock their ships to avoid any sort of negative returns, as this would render the payoff zero. Such an arrangement will create the classic asymmetric payoff structure of a financial option:

$$\Pi(p, f) = \max\left(p(t) - \frac{c}{q \cdot f(t)}, 0\right) \tag{17}$$

3.2 Bioeconomic Black-Scholes PDE

To determine the expected discounted profit of holding this harvesting option over time, we will need to evaluate it under the assumption of uncertain prices. Since we already established that the market price $p(t)$ follows Geometric Brownian Motion back in Chapter 2, we can utilize the fundamental Black-Scholes framework.

The standard Black-Scholes equation for the value of an option $V(S,t)$ is defined as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{18}$$

For us to adapt this PDE into our bioeconomic environment, we will need to carefully substitute our fishery-specific variables into it. For instance, the underlying financial asset S will be replaced by our stochastic market price p. To avoid any sort of mathematical ambiguity with our previously established biological growth rate (r), we can show the continuous financial discount rate (the risk-free rate) as δ.

By assuming that the biological population $f(t)$ will change slowly compared to the rapid, day-to-day volatility of the financial market σ, we can try to treat the extraction cost as a localized constant strike price temporarily. This will of course allow us to define $V(p,t)$ as the expected discounted profit function representing the present value of the opportunity to harvest under stochastic prices. Thus, the Bioeconomic Black-Scholes PDE is structured as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V}{\partial p^2} + \delta p\frac{\partial V}{\partial p} - \delta V = 0 \tag{19}$$

And with our terminal boundary condition at the end of the fishing season ($t = T$), we have:

$$V(p,T) = \max\left(p - \frac{c}{q \cdot f(T)}, 0\right) \tag{20}$$

The purpose of this equation is to anchor the PDE by creating the final deterministic cost value of the fishery right on the very last day of the 90-day season.

3.3 Financial Implications for Fishery Management

By successfully establishing this adapted Partial Differential Equation, we introduce a powerful predictive tool for commercial fleet optimization. The solution to this PDE, $V(p,t)$, represents the definitive financial valuation of the unharvested fish remaining in the water.

This theoretical framework proves that under conditions of high market volatility (a large σ), the ”option value” of delaying the harvest actually increases. If a fleet knows that prices are behaving erratically according to a Geometric Brownian Motion, the Black-Scholes valuation dictates that it may be mathematically optimal to temporarily halt fishing even if the current price is slightly above the extraction cost in order to capitalize on a massive, statistically probable price spike later in the season. By merging the profit payoff function with stochastic price modeling, we have effectively transformed a standard biological harvesting model into a dynamic, risk-adjusted financial portfolio.

3.3.1 Computational Results and Discussion

If we want to visualize the solution to our bioeconomic Partial Differential Equation, we will need to first compute the analytical Black-Scholes formula across the 90-day time domain as well as a varying market price domain. In this case, let's assume that an average mid-season population of $f = 7500$ will fix our biological extraction cost (the strike price) at $X = \$20.00$. We will then apply a daily market volatility of $\sigma = 0.03$ and even provide the Expected Profit Value $V(p, t)$ as a three-dimensional surface.

To compute this, we will use the explicit analytical solution for a European call option. The expected discounted profit can be written as:

$$V(p,t) = p\Phi(d_1) - Xe^{-\delta(T-t)}\Phi(d_2) \tag{21}$$

where $\Phi(d_1)$ and $\Phi(d_2)$ represent the probability factors. $\Phi(d_1)$ predicts if a fisherman ends up making a profit, how much will that catch actually be worth. On the other hand, $\Phi(d_2)$ predicts whether a fisherman actually makes a profit at the end of the 90-day season. Thus, d_1 and d_2 can be defined as:

$$d_1 = \frac{\ln(p/X) + (\delta + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{22}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{23}$$

Furthermore, Φ represents the cumulative distribution function of the standard normal distribution. In this scenario, $T - t$ represents the amount of time that remains in the 90-day season (τ). With the analytical solution, we are now ready to start plotting our 3D model.

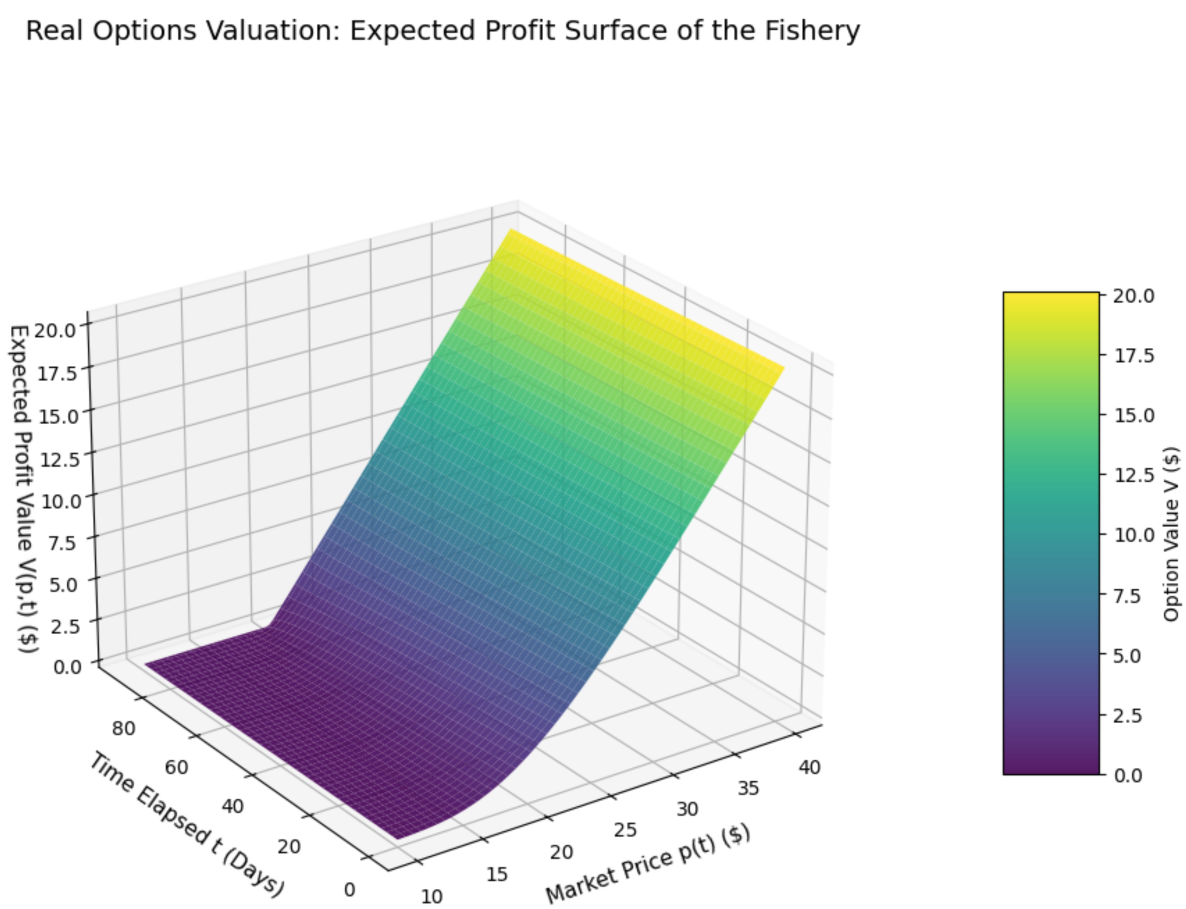

Figure 3.1

From observing this generated 3D surface above, we can see the powerful relationship between the time, stochastic price, and expected profit. The x-axis represents the fluctuating market price $p(t)$, the y-axis tracks the 90 days of the fishing season, and the vertical z-axis represents the expected financial value of holding the option to fish.

As you observe the ”floor” of the model where the market price drops below our \$20.00 extraction cost, we can see that the expected profit value has smoothly approached to zero. Since the fleet is not required to fish at a loss, the overall financial downside of this is strictly limited, visually representing the $\max(p - X, 0)$ payoff condition. On the other hand, as the market price increases toward \$40.00, the surface tends to slope spikes upward in a nearly linear fashion, this appears to indicate a massive expansion in expected profit margins.

However, one of the most critical observations here is presented along the time axis (t). At the very end of the season ($t \approx 90$), we can see that the surface forms a sharp, distinct crease exactly at the \$20.00 strike price. This represents the expiration of the option, where the value is entirely deterministic. But at the beginning of the season ($t = 0$), the surface is distinctly smoothed out and elevated, creating a curved time premium. Thus, this curvature proves our Real Options theory where when there is more time remaining in the season, there is a higher mathematical probability that the stochastic Geometric Brownian Motion will end up causing a sudden, favorable price spike. Therefore, the theoretical value of the unharvested fishery is actually as its highest early in the season, which isn't too surprising in this case scenario as this gives the fishermen more time to strategically delay their maximum harvesting efforts until the market conditions are optimally volatile.

Conclusion

Based on the graphs showing variable costs, we see that even when costs fluctuate, the population of fish still has a significant effect on the profit margin. During the seasonal changes, the sinusoidal nature of the cost function created inverse oscillations in the profit curve. However, the general shape of the profit function was similar to the population curve, as we have already seen from the basic model. When faced with abrupt changes to cost, the profit curve is affected, but continues to follow the trend of the population. For example, even when the cost returned to its baseline level, profit declined until the stock began to regenerate. Changes to price, on the other hand, had a much stronger effect on the profit margin. The stochastic price model was able to completely pull the profit curve from its original behavior. Therefore, market price has a greater impact on profit margin than cost per unit effort and population level. Furthermore, applying the Black-Scholes real options valuation has shown that the expected financial value of the unharvested fishery is at its highest from the beginning of the season, as this makes it mathematically optimal to strategically delay harvesting to capitalize on future market spikes.

Bibliography

[1] Black, F., & Scholes, M. *The Pricing of Options and Corporate Liabilities*. The Journal of Political Economy, Vol. 81, No. 3, May–Jun. 1973, pp. 637–653.

[2] Blank, C. *Seafood inflation outpaces food inflation in January, but winter storms cause shelf-stable sales to soar in US*. SeafoodSource, 18 Feb. 2026. https://www.seafoodsource.com/news/foodservice-retail/seafood-inflation-outpaces-food-inflation-in-january-but-winter-storms-cause-shelf-stable-sales-to-soar

[3] Conrad, J. M. *Resource Economics*. Cambridge University Press, 1999, pp. 35-48.

[4] Duke University Department of Mathematics. *Growth Models, Part 5.2*. https://sites.math.duke.edu/education/postcalc/growth/growth5_2.html

[5] Evans, L. C. *An Introduction to Stochastic Differential Equations*, Version 1.2. Department of Mathematics, UC Berkeley, 2014, pp. 3–6, 78–80, 107–110.

[6] Great Alaska Seafood. *Alaskan Halibut*. https://www.great-alaska-seafood.com/fresh-alaska-halibut.htm

[7] Hicks, R. *The Gordon Schaefer Model*. 19 Oct. 2023. https://rlhick.people.wm.edu/posts/gordon-shaefer-model.html

[8] Hilpisch, Y. *Python for Finance: Mastering Data-Driven Finance*. 2nd ed., O'Reilly Media, 2019, pp. 356–360.

[9] International Pacific Halibut Commission. *Fishery Regulations (2025)*. Document IPHC-2025-FISHR25, Seattle, WA, 2025. https://www.iphc.int/uploads/2025/02/IPHC-Fishery-Regulations-2025-5-Feb-2025.pdf

[10] National Marine Fisheries Service. *Fisheries Economics of the United States, 2022*. U.S. Department of Commerce, NOAA Tech. Memo. NMFS-F/SPO-248A, July 2024. https://media.fisheries.noaa.gov/2024-07/FEUS-2022-v04-0.pdf

[11] NOAA Fisheries. *Setting an Annual Catch Limit*. NOAA. https://www.fisheries.noaa.gov/insight/setting-annual-catch-limit

[12] Owusu, V. *Effect of Rising Fuel Prices on Small-Scale Fisheries Livelihoods and Marine Sustainability in Ghana*. PLOS ONE, Public Library of Science, 13 Jan. 2025. https://journals.plos.org/plosone/article?id=10.1371%2Fjournal.pone.0317260

[13] Shiftlet, A. B., & Shiftlet, G. W. *Introduction to Computational Science*. 2018, pp. 307–314.

[14] United Nations Economic and Social Commission for Western Asia. *Maximum Economic Yield*. https://www.unescwa.org/sd-glossary/maximum-economic-yield

[15] Welch, L. *2026 Pacific Halibut Catches Likely to Be near Last Year's Limits*. Alaska Fish News, 5 Jan. 2026. https://alaskafish.news/01/2026/2026-pacific-halibut-catches-likely-to-be-near-last-years-limits/

Python Model Code

This section provides all of the Python codes that were used to model each simulation throughout this project.

Bioeconomic Fishery Model (Section 1.3)

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

# --- 1. Model Parameters ---
r = 0.05         # Intrinsic growth rate of the fish population
K = 11715        # Carrying capacity of the environment
q = 0.001        # Catchability coefficient (proportionality constant)
p = 40           # Price per unit catch
c = 150          # Cost per unit of fishing effort
days = 90        # Length of the fishing season in days
P0 = [9080]      # Initial fish population

# --- 2. Defining the Effort Function E(t) ---
# The problem specifies effort is a fixed amount at the beginning/end
# and maximum at midseason. A sine wave is a smooth way to model this.
#E_min = 10  # Baseline effort
#E_max = 100 # Maximum midseason effort
E_min = 5
E_max = 50

def E(t):
# Peaks at t = 45 days (midseason)
return E_min + (E_max - E_min) * np.sin(np.pi * t / days)

# --- 3. Differential Equation for Population ---
# dP/dt = r*P*(1 - P/K) - q*P*E(t)
def population_model(t, P):
dP_dt = r * P * (1 - P / K) - q * P * E(t)
return dP_dt

```

```
    # --- 4. Solving the Model ---
    t_span = (0, days)
    t_eval = np.linspace(0, days, 500) # 500 points for a smooth plot

    # Solve the Ordinary Differential Equation (ODE)
    solution = solve_ivp(population_model, t_span, P0, t_eval=t_eval)

    time = solution.t
    population = solution.y[0]
    effort = E(time)

    # --- 5. Economic Model ---
    # Profit per unit catch: pi_catch = p - c / (q * P)
    # Note: As population P drops, the cost to catch a single unit increases.
    profit_per_catch = p - c / (q * population)

    # Total profit rate over time (Profit per catch * Total catch rate)
    catch_rate = q * population * E(time)
    total_profit_rate = profit_per_catch * catch_rate

    # --- 6. Visualization ---
    # Fish Population and Effort
    fig, ax1 = plt.subplots(figsize=(10, 6))

    # Plot Fish Population on primary y-axis
    color1 = 'tab:blue'
    ax1.set_xlabel('Time (Days)')
    ax1.set_ylabel('Fish Population (P)', color=color1)
    ax1.plot(time, population, color=color1, linewidth=2, label='Fish Population')
    ax1.tick_params(axis='y', labelcolor=color1)
    ax1.grid(True, alpha=0.3)

    # Plot Fishing Effort on secondary y-axis
    ax2 = ax1.twinx()
    color2 = 'tab:red'
    ax2.set_ylabel('Fishing Effort (E)', color=color2)
    ax2.plot(time, effort, color=color2, linestyle='-.', linewidth=2, label='Effort E(t)')
    ax2.tick_params(axis='y', labelcolor=color2)

    plt.title('Bioeconomic Model: Fish Population vs. Fishing Effort Over Time')
    fig.tight_layout()
    plt.savefig('population_vs_effort.png')

```

```python
    # Fish Population and Profit
    fig, ax1 = plt.subplots(figsize=(10, 6))

    # Plot Fish Population
    color1 = 'tab:blue'
    ax1.set_xlabel('Time (Days)')
    ax1.set_ylabel('Fish Population (P)', color=color1)
    ax1.plot(time, population, color=color1, linewidth=2, label='Fish Population')
    ax1.tick_params(axis='y', labelcolor=color1)
    ax1.grid(True, alpha=0.3)

    # Create a second y-axis to plot Profit per Unit Catch
    ax2 = ax1.twinx()
    color2 = 'tab:green'
    ax2.set_ylabel('Profit per Unit Catch ($)', color=color2)
    ax2.plot(time, profit_per_catch, color=color2, linestyle='--', linewidth=2, label='Profit / Catch')
    ax2.tick_params(axis='y', labelcolor=color2)

    # Add title and show plot
    plt.title('Bioeconomic Model: Fish Population vs. Profit Margins Over a 90-Day Season')
    fig.tight_layout()
    plt.show()
```

Listing 3.1: Bioeconomic Fishery Model

Periodic Seasonal Costs Model (Section 2.1)

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

# --- 1. Model Parameters ---
r = 0.05          # Intrinsic growth rate of the fish population
K = 11715         # Carrying capacity of the environment
q = 0.001         # Catchability coefficient
p = 40            # Price per unit catch
c = 150           # Cost per unit of fishing effort
days = 90         # Length of the fishing season in days
P0 = [9080]       # Initial fish population

# Effort parameters
#E_min = 10
#E_max = 100
E_min = 5
E_max = 50

# Periodic Cost Parameters: C(t) = C0 + A*sin(omega*t + phi)
C0 = c          # Baseline average cost per unit effort
A = 50            # Amplitude of cost fluctuation
omega = 12 * np.pi / days # Angular frequency (1 full cycle per 90-day
   season)
phi = 0           # Phase shift (starts at baseline cost)

# --- 2. Time-Dependent Functions ---
def E(t):
"""Fishing effort function."""
return E_min + (E_max - E_min) * np.sin(np.pi * t / days)

def C(t):
"""Periodic cost per unit effort function."""
return C0 + A * np.sin(omega * t + phi)

# --- 3. Differential Equation for Population ---
def population_model(t, P):
return r * P * (1 - P / K) - q * P * E(t)

# --- 4. Solving the Model ---
t_span = (0, days)
t_eval = np.linspace(0, days, 500)
solution = solve_ivp(population_model, t_span, P0, t_eval=t_eval)

time = solution.t
```

```
    population = solution.y[0]
    effort = E(time)
    cost_per_effort = C(time)

    # --- 5. Economic Model ---
    # Profit per unit catch: pi_catch = p - C(t) / (q * P)
    profit_per_catch = p - cost_per_effort / (q * population)

    # --- 6. Visualization ---

    # --- Plot 1: Cost per Unit Effort vs. Profit per Unit Catch ---
    fig1, ax1 = plt.subplots(figsize=(10, 6))

    color1 = 'tab:purple'
    ax1.set_xlabel('Time (Days)')
    ax1.set_ylabel('Cost per Unit Effort ($)', color=color1)
    ax1.plot(time, cost_per_effort, color=color1, linewidth=2, label='Cost C(t)')
    ax1.tick_params(axis='y', labelcolor=color1)
    ax1.grid(True, alpha=0.3)

    ax2 = ax1.twinx()
    color2 = 'tab:green'
    ax2.set_ylabel('Profit per Unit Catch ($)', color=color2)
    ax2.plot(time, profit_per_catch, color=color2, linestyle='--', linewidth=2, label='Profit Margin')
    ax2.tick_params(axis='y', labelcolor=color2)

    plt.title('Bioeconomic Model: Cost vs. Profit per Catch')
    fig1.tight_layout()
    plt.show()

    # --- Plot 2: Fish Population vs. Profit per Unit Catch ---
    fig2, ax3 = plt.subplots(figsize=(10, 6))

    color3 = 'tab:blue'
    ax3.set_xlabel('Time (Days)')
    ax3.set_ylabel('Fish Population (P)', color=color3)
    ax3.plot(time, population, color=color3, linewidth=2, label='Fish Population')
    ax3.tick_params(axis='y', labelcolor=color3)
    ax3.grid(True, alpha=0.3)

    ax4 = ax3.twinx()
    color4 = 'tab:green'
    ax4.set_ylabel('Profit per Unit Catch ($)', color=color4)
```

```
ax4.plot(time, profit_per_catch, color=color4, linestyle='--', linewidth=2, label='Profit Margin')
ax4.tick_params(axis='y', labelcolor=color4)

plt.title('Bioeconomic Model: Fish Population vs. Profit per Catch')
fig2.tight_layout()
plt.show()
```

Listing 3.2: Periodic Seasonal Costs Model

Abrupt Cost Changes Model (Section 2.2)

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

# --- 1. Model Parameters ---
r = 0.05         # Intrinsic growth rate of the fish population
K = 11715        # Carrying capacity of the environment
q = 0.001        # Catchability coefficient
p = 40           # Price per unit catch
days = 90        # Length of the fishing season in days
P0 = [9080]      # Initial fish population

# Effort parameters
#E_min = 10
#E_max = 100
E_min = 5
E_max = 50

# Abrupt Cost Parameters for Step Function
C_normal = 150     # Baseline cost per unit effort
C_high = 300       # Spiked cost per unit effort
t_start_spike = 30 # Day the cost spikes
t_end_spike = 60   # Day the cost returns to normal

# --- 2. Time-Dependent Functions ---
def E(t):
"""Fishing effort function."""
return E_min + (E_max - E_min) * np.sin(np.pi * t / days)

def C(t):
"""Piecewise cost per unit effort function."""
# np.where handles both scalar (from ODE solver) and array inputs (for plotting)
```

```python
return np.where((t >= t_start_spike) & (t <= t_end_spike), C_high, C_normal)

# --- 3. Differential Equation for Population ---
def population_model(t, P):
return r * P * (1 - P / K) - q * P * E(t)

# --- 4. Solving the Model ---
t_span = (0, days)
t_eval = np.linspace(0, days, 500)
solution = solve_ivp(population_model, t_span, P0, t_eval=t_eval)

time = solution.t
population = solution.y[0]
cost_per_effort = C(time)

# --- 5. Economic Model ---
# Profit per unit catch: pi_catch = p - C(t) / (q * P)
profit_per_catch = p - cost_per_effort / (q * population)

# --- 6. Visualization ---
# --- Plot 1: Cost per Unit Effort vs. Profit per Unit Catch ---
fig1, ax1 = plt.subplots(figsize=(10, 6))

color1 = 'tab:purple'
ax1.set_xlabel('Time (Days)')
ax1.set_ylabel('Cost per Unit Effort ($)', color=color1)
ax1.plot(time, cost_per_effort, color=color1, linewidth=2, label='Cost C(t)')
ax1.tick_params(axis='y', labelcolor=color1)
ax1.grid(True, alpha=0.3)

ax2 = ax1.twinx()
color2 = 'tab:green'
ax2.set_ylabel('Profit per Unit Catch ($)', color=color2)
ax2.plot(time, profit_per_catch, color=color2, linestyle='--', linewidth=2, label='Profit Margin')
ax2.tick_params(axis='y', labelcolor=color2)

plt.title('Bioeconomic Model: Abrupt Cost Changes vs. Profit')
fig1.tight_layout()
plt.show()

# --- Plot 2: Fish Population vs. Profit per Unit Catch ---
fig2, ax3 = plt.subplots(figsize=(10, 6))

color3 = 'tab:blue'
```

```
    ax3.set_xlabel('Time (Days)')
    ax3.set_ylabel('Fish Population (P)', color=color3)
    ax3.plot(time, population, color=color3, linewidth=2, label='Fish
       Population')
    ax3.tick_params(axis='y', labelcolor=color3)
    ax3.grid(True, alpha=0.3)

    ax4 = ax3.twinx()
    color4 = 'tab:green'
    ax4.set_ylabel('Profit per Unit Catch ($)', color=color4)
    ax4.plot(time, profit_per_catch, color=color4, linestyle='--', linewidth
       =2, label='Profit Margin')
    ax4.tick_params(axis='y', labelcolor=color4)

    plt.title('Bioeconomic Model: Fish Population vs. Profit under Abrupt
       Costs')
    fig2.tight_layout()
    plt.show()
```

Listing 3.3: Abrupt Cost Changes Model

Stochastic Market Price Model (Section 2.3)

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

# --- 1. Biological & Harvesting Parameters ---
r = 0.05
K = 11715
q = 0.001
c = 150
days = 90
P0 = [9080]

E_min = 5
E_max = 50

def E(t):
return E_min + (E_max - E_min) * np.sin(np.pi * t / days)

# --- 2. Differential Equation for Population ---
def population_model(t, P):
return r * P * (1 - P / K) - q * P * E(t)

```

```
    # Solve Biological Model (Only needs to be solved once)
    N_steps = 500
    t_span = (0, days)
    t_eval = np.linspace(0, days, N_steps)
    solution = solve_ivp(population_model, t_span, P0, t_eval=t_eval)

    time = solution.t
    population = solution.y[0]

    # --- 3. Geometric Brownian Motion Parameters ---
    p0 = 40
    mu = 0.02
    sigma = 0.15
    dt = days / (N_steps - 1)

    # --- 4. Plotting Two 3x2 Grids ---
    # Set up two separate figures
    fig1, axes1 = plt.subplots(3, 2, figsize=(15, 12), constrained_layout=True)
    fig2, axes2 = plt.subplots(3, 2, figsize=(15, 12), constrained_layout=True)

    # Flatten the 3x2 array of axes for easy iteration
    axes1 = axes1.flatten()
    axes2 = axes2.flatten()

    # We are intentionally NOT using a random seed here so every run is unique
    for i in range(6):
    # Generate independent Brownian motion path
    dW = np.random.normal(0, np.sqrt(dt), N_steps)
    W = np.cumsum(dW)
    W[0] = 0

    # Calculate stochastic price p(t)
    price = p0 * np.exp((mu - 0.5 * sigma**2) * time + sigma * W)

    # Calculate resulting profit per catch
    profit_per_catch = price - c / (q * population)

    # -----------------------------------------------------------
    # Plotting Figure 1: Market Price vs. Profit
    # -----------------------------------------------------------
    ax1_price = axes1[i]
    color_price = 'tab:orange'
    ax1_price.plot(time, price, color=color_price, linewidth=2)
    ax1_price.set_ylabel('Market Price ($)', color=color_price)
```

```
    ax1_price.tick_params(axis='y', labelcolor=color_price)
    ax1_price.set_xlabel('Time (Days)')
    ax1_price.grid(True, alpha=0.3)

    ax1_profit = ax1_price.twinx()
    color_profit = 'tab:green'
    ax1_profit.plot(time, profit_per_catch, color=color_profit, linestyle='--', linewidth=2)
    ax1_profit.set_ylabel('Profit Margin ($)', color=color_profit)
    ax1_profit.tick_params(axis='y', labelcolor=color_profit)
    ax1_price.set_title(f'Realization {i+1}: Price vs. Profit Margin')

    # ---------------------------------------------------------
    # Plotting Figure 2: Fish Population vs. Profit
    # ---------------------------------------------------------
    ax2_pop = axes2[i]
    color_pop = 'tab:blue'
    ax2_pop.plot(time, population, color=color_pop, linewidth=2)
    ax2_pop.set_ylabel('Fish Population (P)', color=color_pop)
    ax2_pop.tick_params(axis='y', labelcolor=color_pop)
    ax2_pop.set_xlabel('Time (Days)')
    ax2_pop.grid(True, alpha=0.3)

    ax2_profit = ax2_pop.twinx()
    ax2_profit.plot(time, profit_per_catch, color=color_profit, linestyle='--', linewidth=2)
    ax2_profit.set_ylabel('Profit Margin ($)', color=color_profit)
    ax2_profit.tick_params(axis='y', labelcolor=color_profit)
    ax2_pop.set_title(f'Realization {i+1}: Population vs. Profit Margin')

    # Final layout adjustments for Figure 1
    fig1.suptitle('Bioeconomic Model: 6 Realizations of Stochastic Market Price vs. Profit', fontsize=16)
    plt.show()

    # Final layout adjustments for Figure 2
    fig2.suptitle('Bioeconomic Model: 6 Realizations of Fish Population vs. Profit', fontsize=16)
    plt.show()
```

Listing 3.4: Stochastic Market Price Model

Real Options Valuation: Black-Scholes 3D Surface (Section 3.3.1)

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

# --- 1. Bioeconomic & Financial Parameters ---
c = 150              # Operational cost per unit effort
q = 0.001            # Catchability coefficient
f_avg = 7500         # Average mid-season fish population
X = c / (q * f_avg) # Strike Price (Extraction Cost) = $20.00

days = 90            # Length of the season in days
delta = 0.05 / 365   # Daily risk-free discount rate (Assuming 5% annual)
sigma = 0.03         # Daily market volatility

# --- 2. Grid for Price (p) and Time (t) ---
# Price ranges from $10 to $40
p = np.linspace(10, 40, 100)
# Time ranges from Day 0 to Day 89.9 (Avoid exactly 90 to prevent
   division by zero)
t = np.linspace(0, 89.9, 100)
P, T_mesh = np.meshgrid(p, t)

# Time to maturity (end of the season)
tau = days - T_mesh

# --- 3. The Analytical Black-Scholes Formula ---
d1 = (np.log(P / X) + (delta + 0.5 * sigma**2) * tau) / (sigma * np.sqrt
   (tau))
d2 = d1 - sigma * np.sqrt(tau)

# Expected discounted profit (Option Value V)
V = P * norm.cdf(d1) - X * np.exp(-delta * tau) * norm.cdf(d2)

# --- 4. 3D Visualization ---
fig = plt.figure(figsize=(12, 8))
ax = fig.add_subplot(111, projection='3d')

# Plotting the surface
surf = ax.plot_surface(P, T_mesh, V, cmap='viridis', edgecolor='none',
   alpha=0.9)

ax.set_xlabel('Market Price p(t) ($)', fontsize=11, labelpad=10)
ax.set_ylabel('Time Elapsed t (Days)', fontsize=11, labelpad=10)
ax.set_zlabel('Expected Profit Value V(p,t) ($)', fontsize=11, labelpad
   =10)
```

```
    ax.set_title('Real Options Valuation: Expected Profit Surface of the
       Fishery', fontsize=14)

    fig.colorbar(surf, shrink=0.5, aspect=5, label='Option Value V ($)')

    # Viewing angle
    ax.view_init(elev=25, azim=-125)
    ax.set_box_aspect(aspect=None, zoom=0.85)
    plt.subplots_adjust(left=0.0, right=0.95, top=0.95, bottom=0.0)
    plt.tight_layout(pad=3.0)
    plt.show()
```
Listing 3.5: Real Options Valuation: Black-Scholes 3D Surface